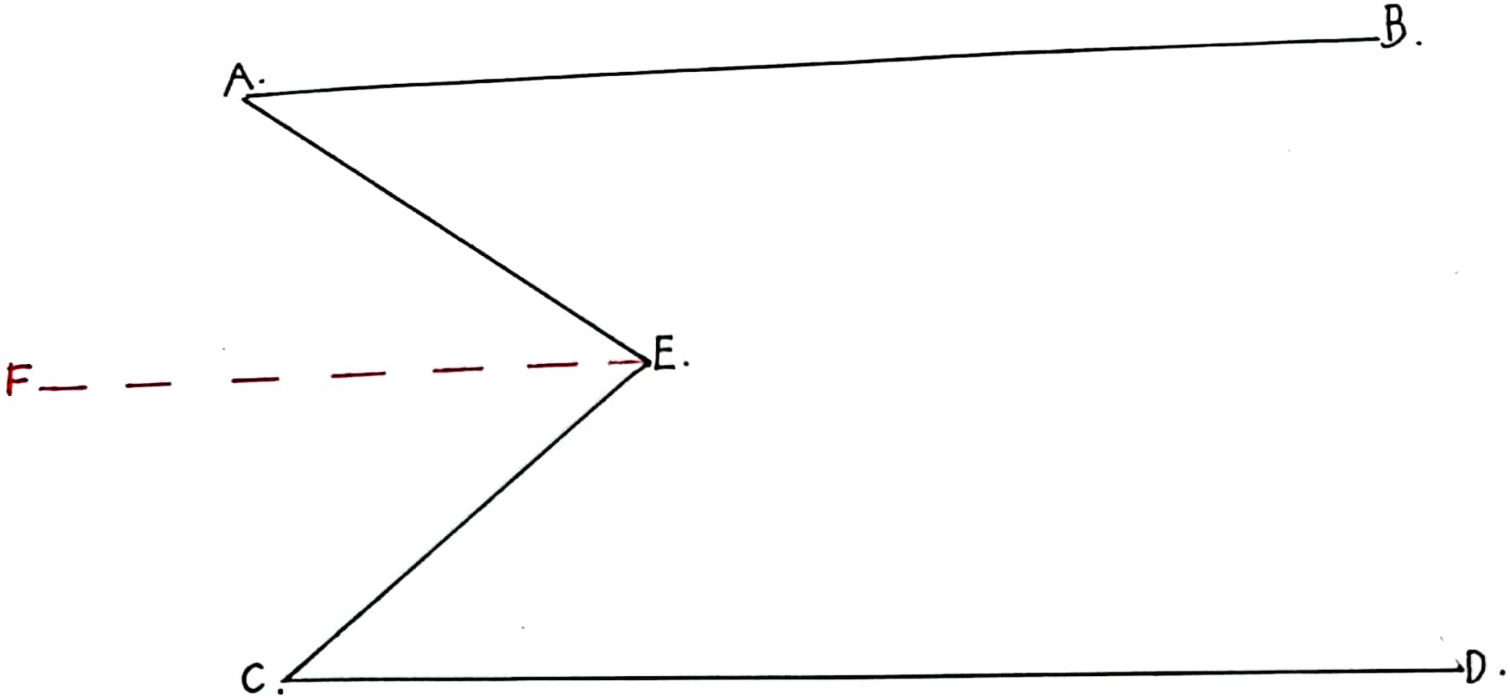


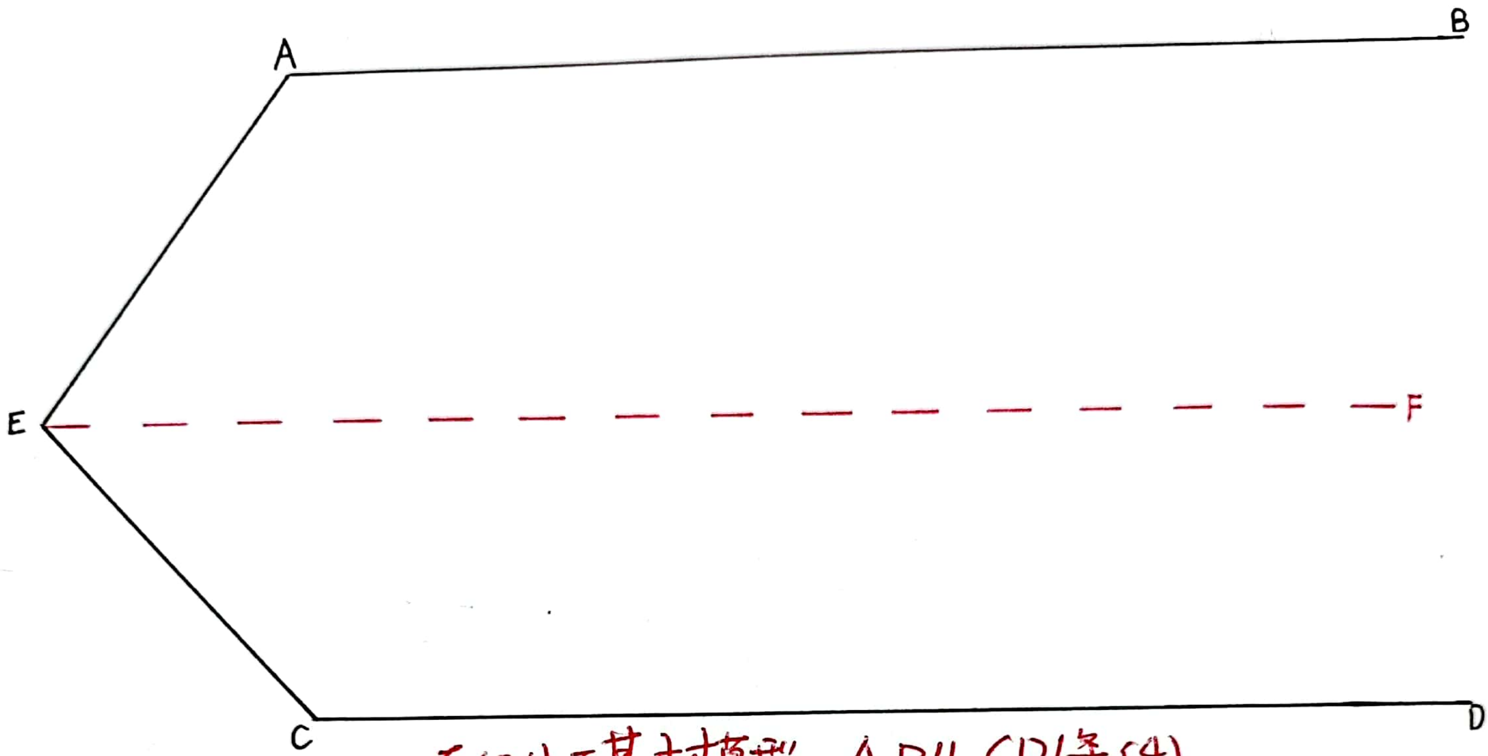
条件 AB//CD. 结论: $\angle A+\angle C=\angle AEC$

证明: 解: 过E点作辅助线EF//AB

$\because EF // AB$

$\therefore \angle A=\angle ACF$(两直线平行,内错角相等)

$\therefore \angle A+\angle C=\angle AEC$



平行线下基本模型 AB∥CD(条件)

结论：$\angle A+\angle C+\angle AEC=360^\circ$

证明：~~解~~：过E点作辅助线EF∥AB

$\because AB\parallel EF$(已知)

$\therefore \angle A+\angle AEF=180^\circ$(两直线平行，同旁内角互补)

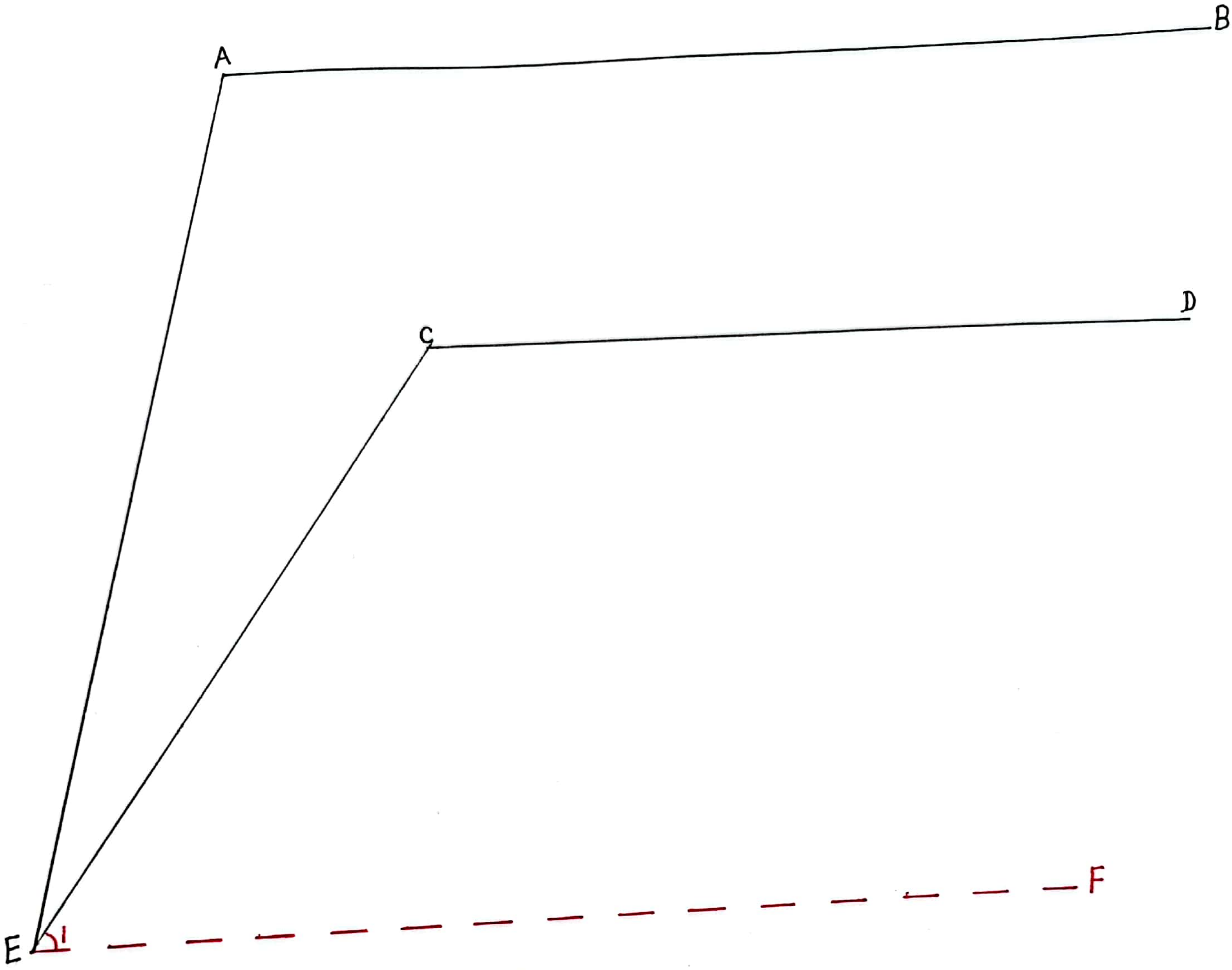
$\because CD\parallel EF$(已知)

$\therefore \angle C+\angle FEC=180^\circ$(两直线平行，同旁内角互补)

$\because \angle A+\angle AEF=180^\circ$

$\angle C+\angle FEC=180^\circ$

$\therefore \angle A+\angle C+\angle AEC=360^\circ$



结论:$\angle A+\angle CEA=\angle C$

证明:~~解~~:过E点作辅助线$EF\parallel CD$

$\because EF\parallel CD$

$\therefore \angle C+\angle 1=180^\circ$(两直线平行,同旁内角互补)

$\therefore \angle C=180^\circ-\angle 1$

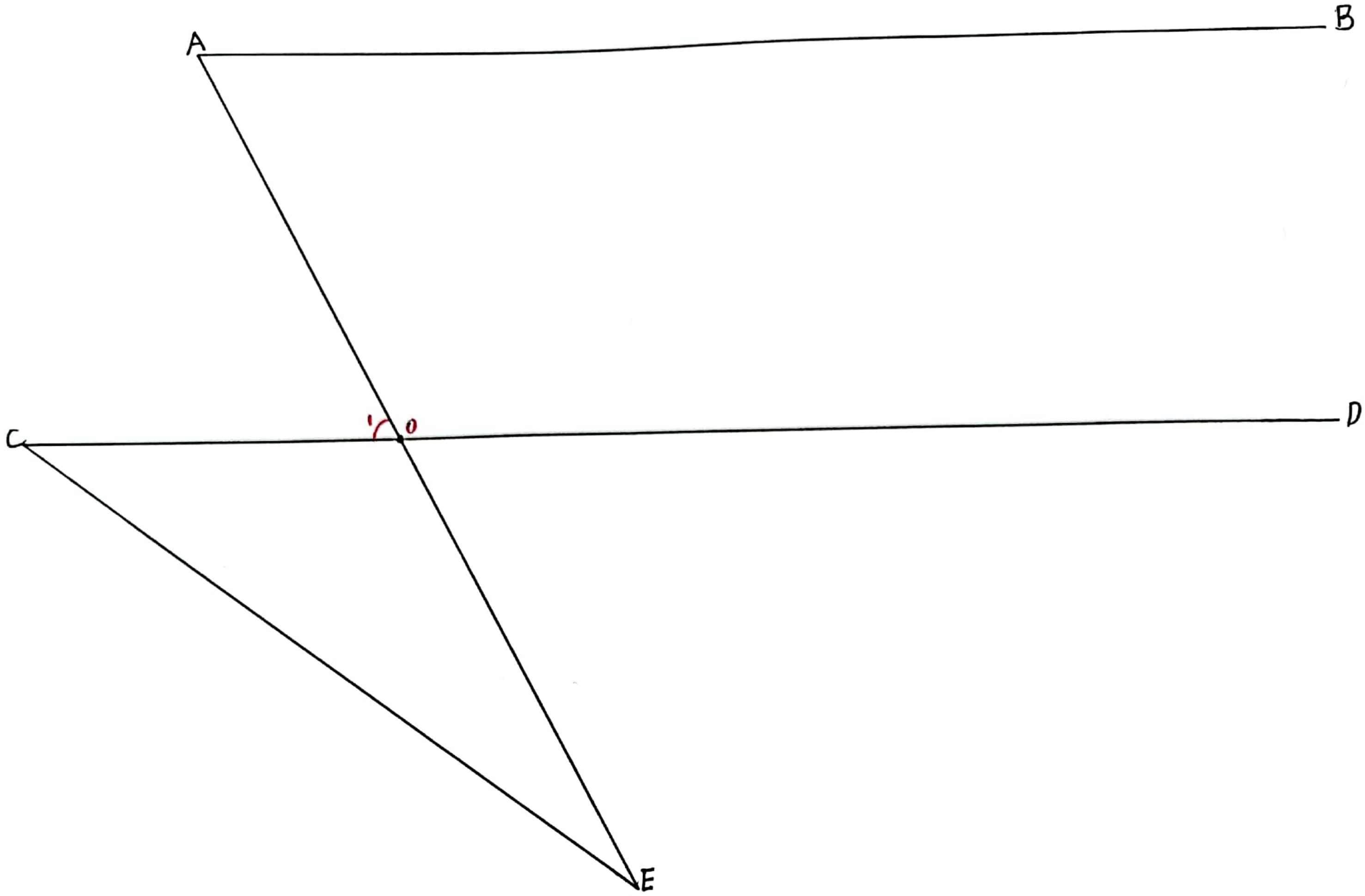
$\because AB\parallel CD$

$\therefore AB\parallel EF$

$\therefore \angle AEC+\angle A+\angle 1=180^\circ$(两直线平行,同旁内角互补)

$\therefore \angle AEC+\angle A=180^\circ-\angle 1$

$\therefore \angle A+\angle AEC=\angle C$



结论：$\angle C+\angle CEA=\angle A$

证明：解：∵三角形的一个外角等于两个不相邻的内角之和

∴$\angle C+\angle CEA=\angle 1$

∵$AB\parallel CD$

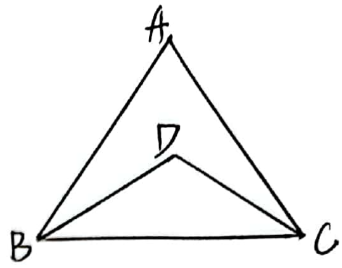
∴$\angle A=\angle 1$（两直线平行，内错角相等）

∴$\angle C+\angle CEA=\angle A$（等量代换）

结论：$\angle EAD=\frac{1}{2}(\angle C-\angle B)$

① 两个内角角平分线夹角

结论：$\angle D=90^\circ+\frac{1}{2}\angle A$



证明：∵在$\triangle ABC$中 ∴$\angle A+\angle ABC+\angle ACB=180^\circ$

∴$\angle ABC+\angle ACB=180^\circ-\angle A$

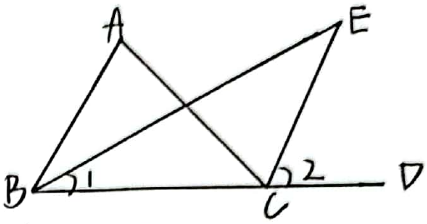
∵BD，CD分别为$\angle ABC$，$\angle ACB$的角平分线

∴$\angle DBC+\angle DCB=\frac{1}{2}(\angle ABC+\angle ACB)=\frac{1}{2}(180^\circ-\angle A)=90^\circ-\frac{1}{2}\angle A$

∵在$\triangle BDC$中 ∴$\angle D=180^\circ-(\angle DBC+\angle DCB)=180^\circ-(90-\frac{1}{2}\angle A)=90^\circ+\frac{1}{2}\angle A$

② 一外一内角平分线夹角

结论：$\angle A=2\angle E$



证明：∵$\angle 2$为$\triangle BEC$的外角 ∴$\angle E=\angle 2-\angle 1$

∵$\angle ACD$为$\triangle ABC$的外角 ∴$\angle A=\angle ACD-\angle ABC$

∵CE为$\angle ACD$的角平分线，BE为$\angle ABC$的角平分线 ∴$\angle ACD=2\angle 2$ $\angle ABC=2\angle 1$

∴$\angle A=2(\angle 2-\angle 1)$

∴$\angle A=2\angle E$

③ 两个外角角平分线的夹角

结论：$\angle A+2\angle F=180^\circ$ / $\angle F=90^\circ-\frac{1}{2}\angle A$

证明：∵$\angle DBC$为$\triangle ABC$的外角 ∴$\angle A+\angle ACB=\angle DBC$

∵$\angle BCE$为$\triangle ABC$的外角 ∴$\angle A+\angle ABC=\angle BCE$

∵在$\triangle ABC$中 ∴$\angle A+\angle ABC+\angle ACB=180^\circ$

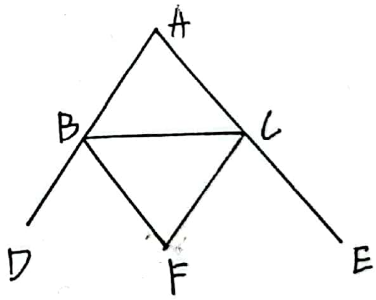
∴$\angle DBC+\angle BCE=2\angle A+\angle ACB+\angle ABC$

$=\angle A+180^\circ$

∵BF，CF分别平分$\angle DBC$，$\angle BCE$

∴$\angle BCF=\frac{1}{2}\angle BCE$

$\angle CBF=\frac{1}{2}\angle DBC$



∵在$\triangle BFC$中 ∴$\angle F+\angle BCF+\angle CBF=180^\circ$

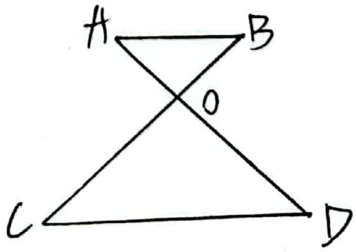
$\angle F+\frac{1}{2}\angle BCE+\frac{1}{2}\angle DBC=180^\circ$

∴$\angle F+\frac{\angle A+180}{2}=180^\circ$

∴$2\angle F+\angle A=180^\circ$

④ "8"型

结论：$\angle A+\angle B=\angle C+\angle D$



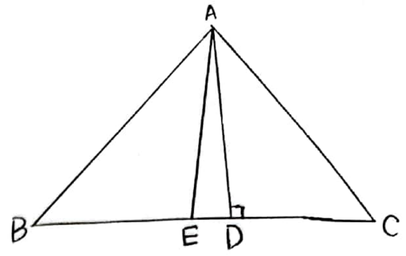
证明：∵在$\triangle ABO$中 ∴$\angle B+\angle A+\angle BOA=180^\circ$

又∵在$\triangle COD$中 ∴$\angle C+\angle D+\angle COD=180^\circ$

又∵$\angle BOA=\angle COD$（对顶角相等） ∴$\angle A+\angle B=\angle C+\angle D$

⑤

结论：$\angle EAD=\frac{1}{2}(\angle C-\angle B)$



证明：∵在△ABC中

$\therefore \angle A+\angle B+\angle C=180^\circ$

$\therefore \angle A=180^\circ-\angle B-\angle C$

∵AE平分∠BAC

$\therefore \angle EAC=\frac{1}{2}\angle BAC=\frac{180^\circ-\angle B-\angle C}{2}$

$\angle DAC=180^\circ-90^\circ-\angle C=90^\circ-\angle C$

∵AD是△ABC的高

$\therefore AD\perp BC$

$\therefore \angle ADC=90^\circ$

$\therefore \angle EAD=\angle EAC-\angle DAC$

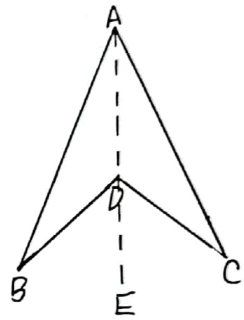
$=\frac{180^\circ-\angle B-\angle C}{2}-(90^\circ-\angle C)$

$=90^\circ-\frac{1}{2}\angle B-\frac{1}{2}\angle C-90^\circ+\angle C$

$=\frac{1}{2}(\angle C-\angle B)$

⑥

结论：$\angle BDC=\angle C+\angle B+\angle BAC$



证明：连接AD到点E

∵∠BDE是△ABD的外角

$\therefore \angle BDE=\angle B+\angle BAD$

∵∠CDE是△ACD的外角

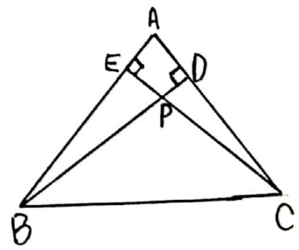
$\therefore \angle CDE=\angle C+\angle CAD$

$\therefore \angle BDE+\angle CDE=\angle C+\angle B+\angle BAD+\angle CAD$

$\therefore \angle BDC=\angle C+\angle B+\angle BAC$

⑦

结论：$\angle BPC=180^\circ-\angle A$



证明：∵EC是△ABC的高

$\therefore EC\perp AB$

$\therefore \angle CEA=90^\circ$

∵BD是△ABC的高

$\therefore BD\perp AC$

$\therefore \angle BDA=90^\circ$

∵在四边形AEPD中

∴∠EPD=四边形AEPD内角和为360°

$\therefore \angle EPD=360^\circ-90^\circ-90^\circ-\angle A$

$=180^\circ-\angle A$

∵∠BPC=∠EPD(对顶角相等)

$\therefore \angle BPC=180^\circ-\angle A$

结论：$\angle B=\angle P$

证明：延长DA、CE交于点P

$\because$ AD是$\triangle ABC$的高

$\therefore AD\perp BC$

$\therefore \angle BAD=90^\circ$，$\angle ADC=90^\circ$

$\because$ CE是$\triangle ABC$的高

$\therefore CE\perp AB$

$\therefore \angle CEB=90^\circ$，$\angle PEB=90^\circ$

$\because$ 在$\triangle BAD$中

$\therefore \angle B+\angle ADB+\angle BAD=180^\circ$

$\therefore \angle B=180^\circ-\angle ADB-\angle BAD$

$\because$ 在$\triangle PEA$中

$\therefore \angle P+\angle PEA+\angle PAE=180^\circ$

$\therefore \angle P=180^\circ-\angle PEA-\angle PAE$

$\because \angle PAE=\angle BAD$（对顶角相等）

又$\because \angle PEA=\angle BDA$

$\therefore \angle B=\angle P$